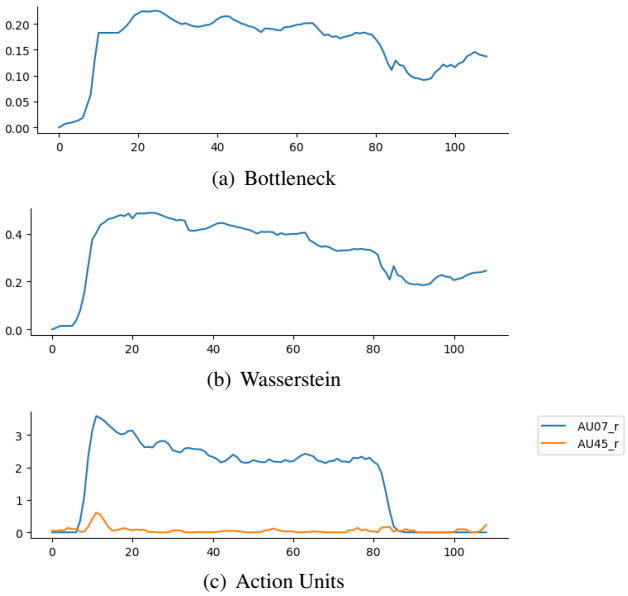
Supplement: AffectiveTDA: Using Topological Data Analysis to Improve Analysis and Explainability in Affective Computing

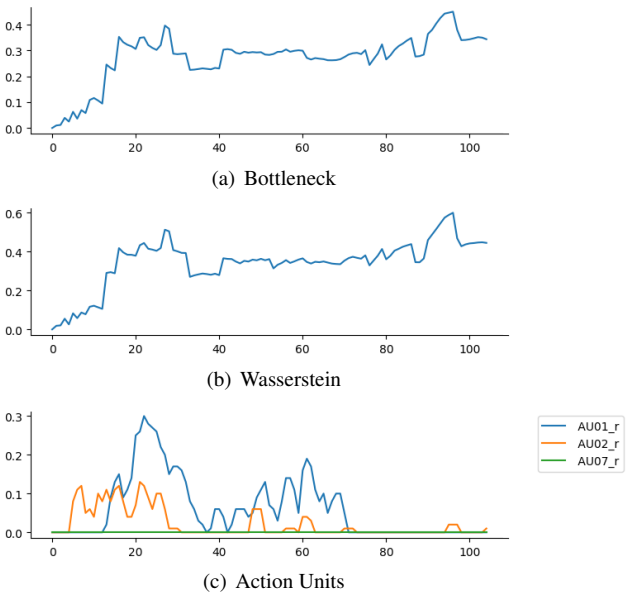
Hamza Elhamdadi, Shaun Canavan, and Paul Rosen

Additional Examples of Relative Distance Topology and Action Units (AUs)



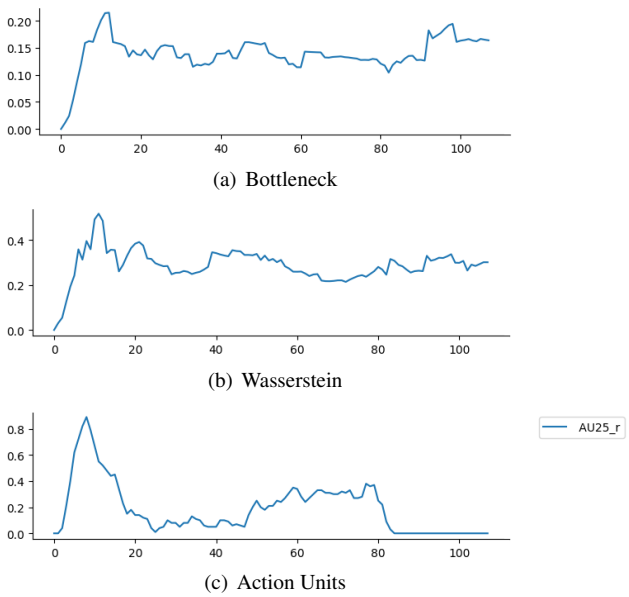
(a) Bottleneck (b) Wasserstein (c) Action Units

Fig. 1. F002 *Anger* using eyes, eyebrows, nose, and mouth



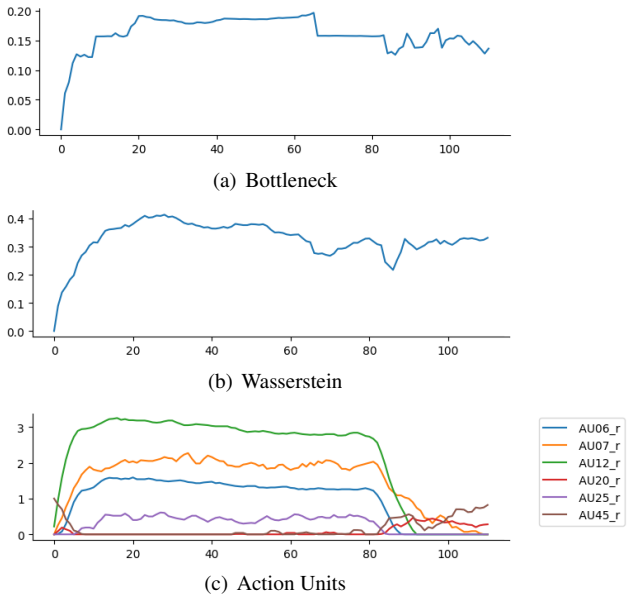
(a) Bottleneck (b) Wasserstein (c) Action Units

Fig. 2. F002 *Fear* using eyes, eyebrows, nose, and mouth



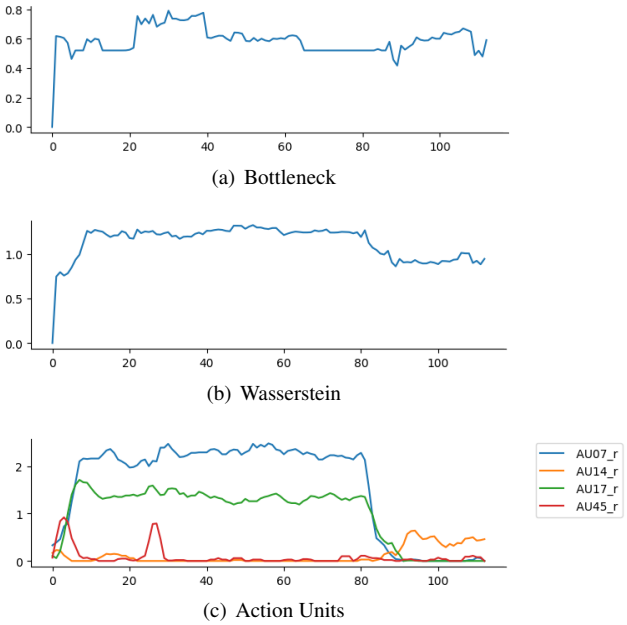
(a) Bottleneck (b) Wasserstein (c) Action Units

Fig. 3. F002 *Disgust* using eyes, eyebrows, nose, and mouth



(a) Bottleneck (b) Wasserstein (c) Action Units

Fig. 4. F002 *Happiness* using eyes, eyebrows, nose, and mouth

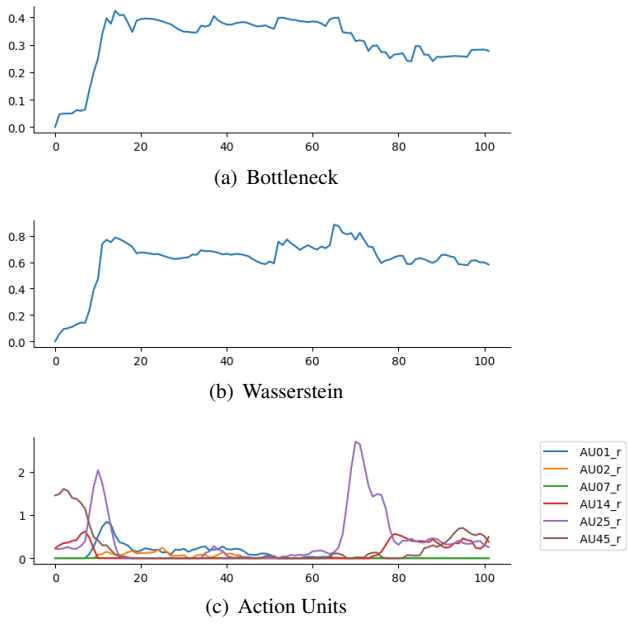


(a) Bottleneck

(b) Wasserstein

(c) Action Units

Fig. 5. F002 *Sadness* using eyes, eyebrows, nose, and mouth

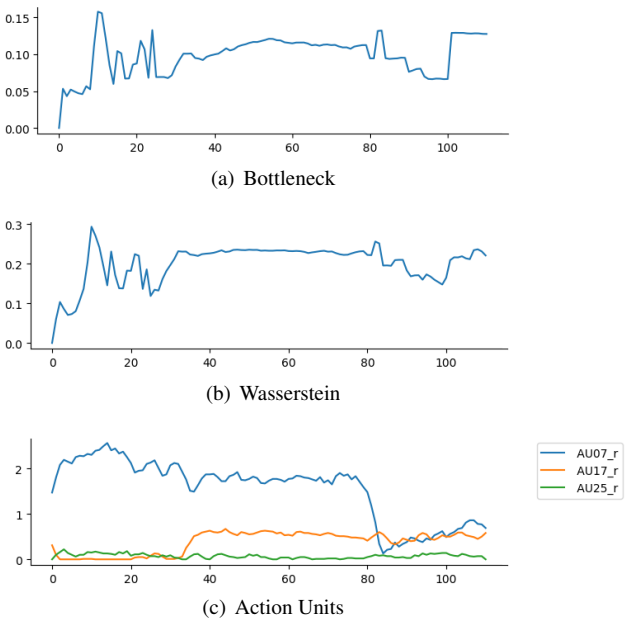


(a) Bottleneck

(b) Wasserstein

(c) Action Units

Fig. 7. F002 *Surprise* using eyes, eyebrows, nose, and mouth

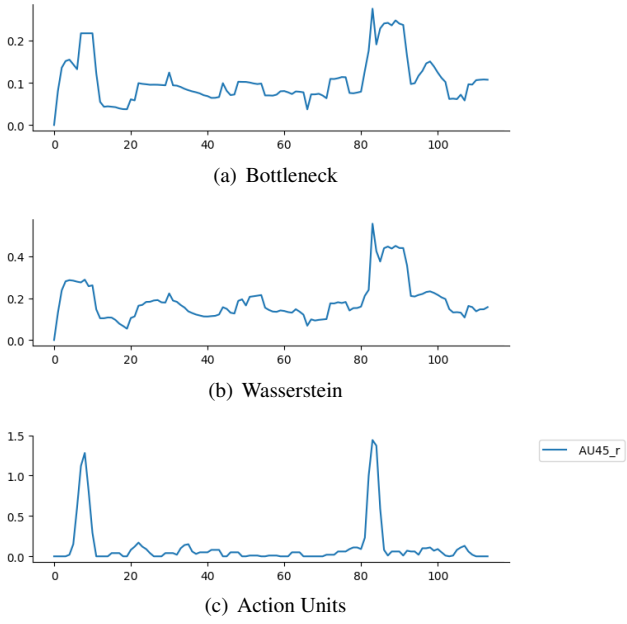


(a) Bottleneck

(b) Wasserstein

(c) Action Units

Fig. 6. M002 *Anger* using eyes, eyebrows, nose, and mouth

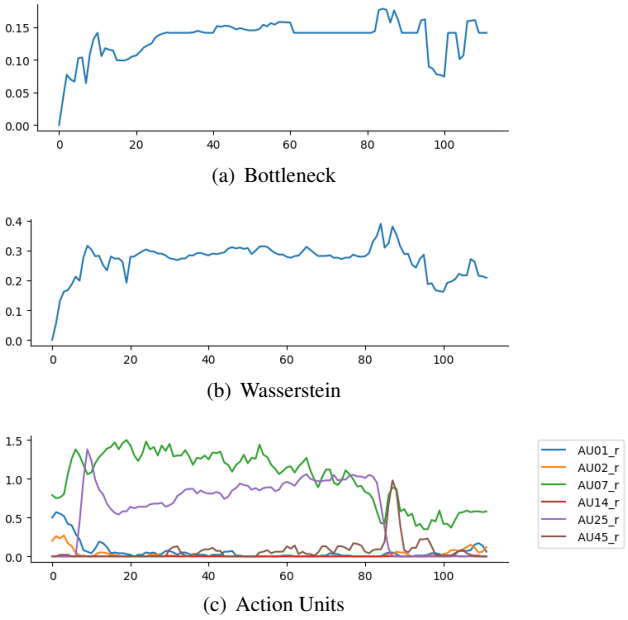


(a) Bottleneck

(b) Wasserstein

(c) Action Units

Fig. 8. M002 *Disgust* using eyes, eyebrows, nose, and mouth

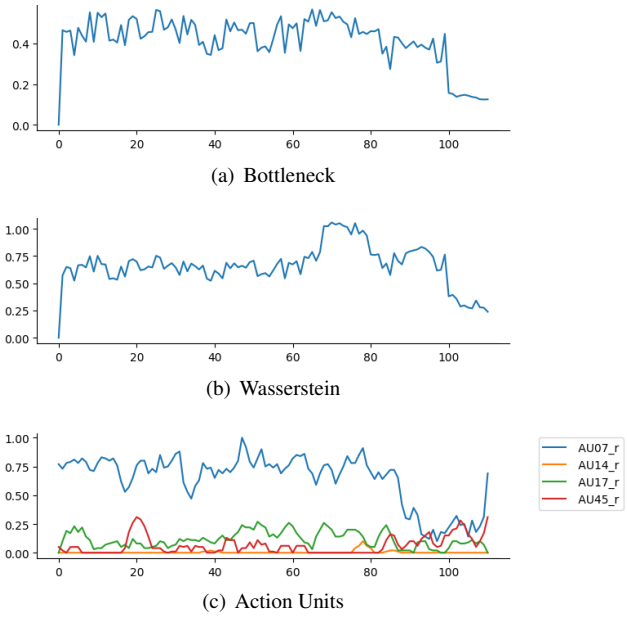


(a) Bottleneck

(b) Wasserstein

(c) Action Units

Fig. 9. M002 *Fear* using eyes, eyebrows, nose, and mouth

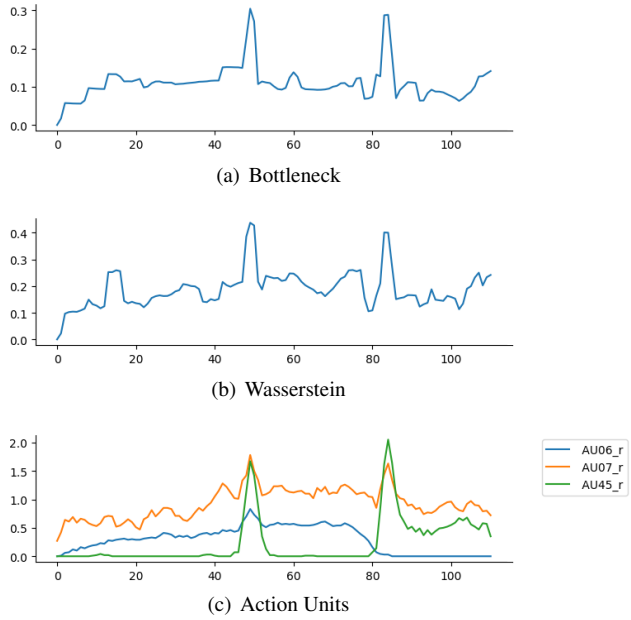


(a) Bottleneck

(b) Wasserstein

(c) Action Units

Fig. 10. M002 *Sadness* using eyes, eyebrows, nose, and mouth

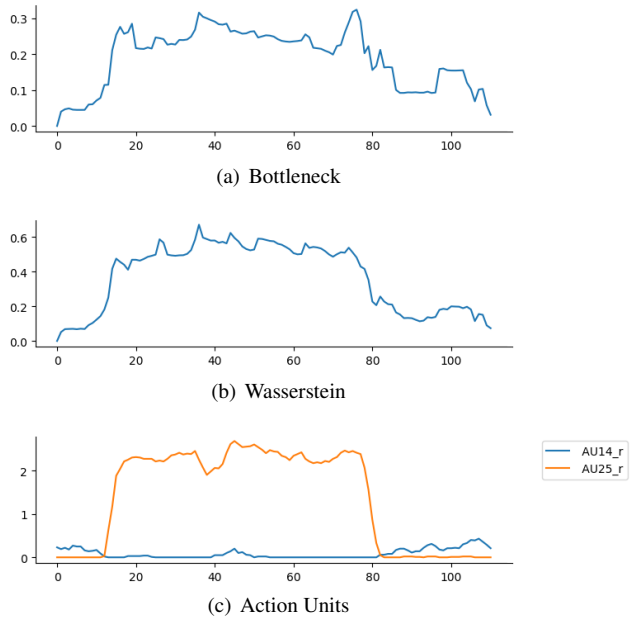


(a) Bottleneck

(b) Wasserstein

(c) Action Units

Fig. 11. M002 *Happiness* using eyes, eyebrows, nose, and mouth



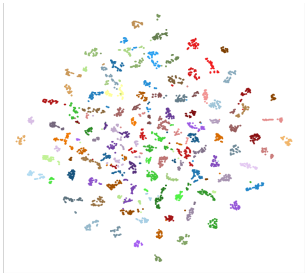
(a) Bottleneck

(b) Wasserstein

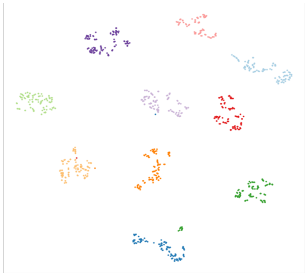
(c) Action Units

Fig. 12. M002 *Surprise* using eyes, eyebrows, nose, and mouth

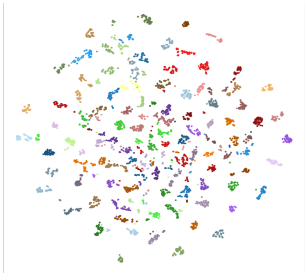
ADDITIONAL EXAMPLES COMPARING AND DIFFERENTIATING INDIVIDUALS



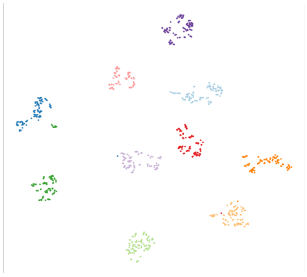
(a) *Anger* perplexity: 30 (full)



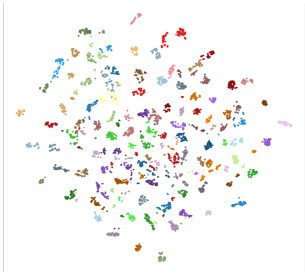
(b) *Anger* perplexity: 30 (10 subj.)



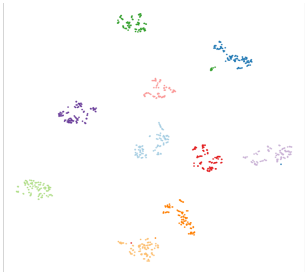
(c) *Anger* perplexity: 40 (full)



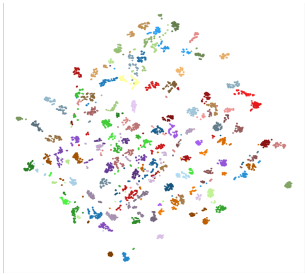
(d) *Anger* (10 subj.) Perplexity=40



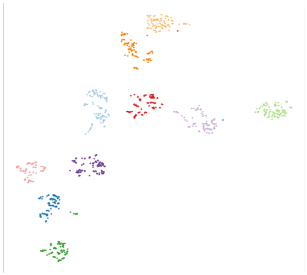
(e) *Anger* perplexity: 50 (full)



(f) *Anger* perplexity: 50 (10 subj.)



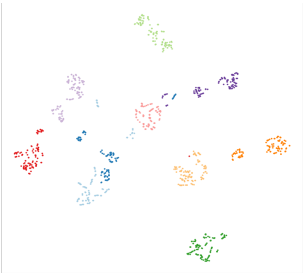
(g) *Anger* perplexity: 100 (full)



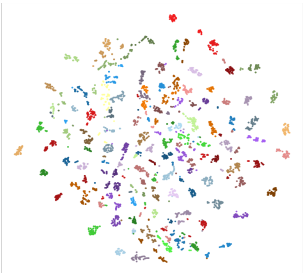
(h) *Anger* perplexity: 100 (10 subj.)

Fig. 13. t-SNE clustering of individual topological data for *Anger* emotion at different perplexities.

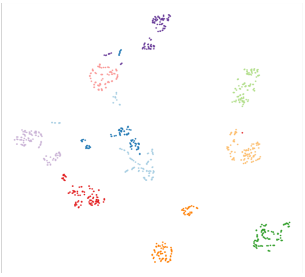
(a) *Disgust* perplexity: 30 (full)



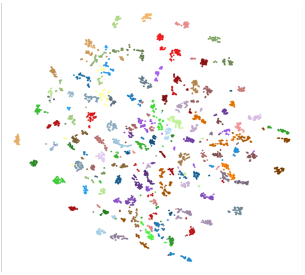
(b) *Disgust* perplexity: 30 (10 subj.)



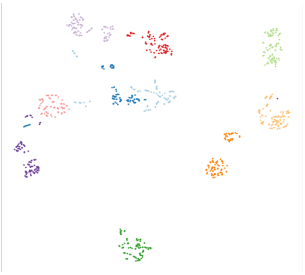
(c) *Disgust* perplexity: 40 (full)



(d) *Disgust* perplexity: 40 (10 subj.)



(e) *Disgust* perplexity: 50 (full)



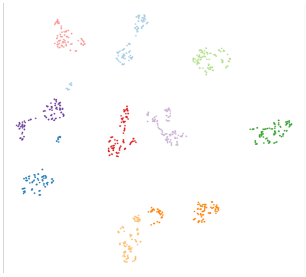
(f) *Disgust* perplexity: 50 (10 subj.)

(g) *Disgust* perplexity: 100 (full)

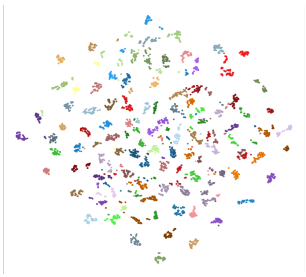
(h) *Disgust* perplexity: 100 (10 subj.)

Fig. 14. t-SNE clustering of individual topological data for *Disgust* emotion at different perplexities.

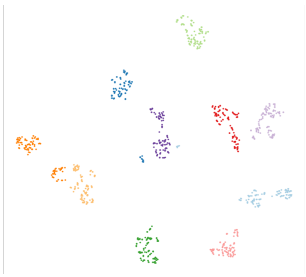
(a) *Fear* perplexity: 30 (full)



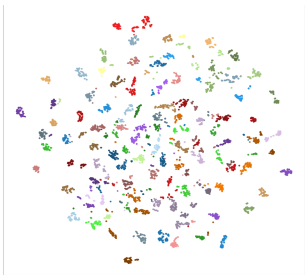
(b) *Fear* perplexity: 30 (10 subj.)



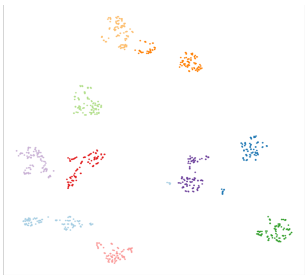
(c) *Fear* perplexity: 40 (full)



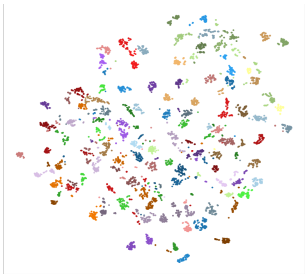
(d) *Fear* perplexity: 40 (10 subj.)



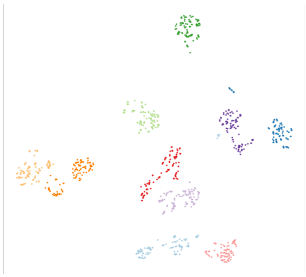
(e) *Fear* perplexity: 50 (full)



(f) *Fear* perplexity: 50 (10 subj.)

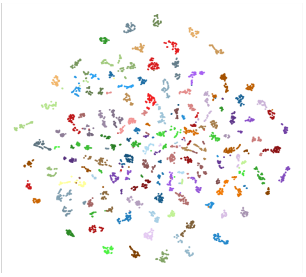


(g) *Fear* perplexity: 100 (full)

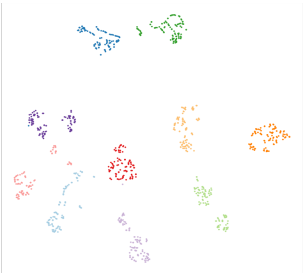


(h) *Fear* perplexity: 100 (10 subj.)

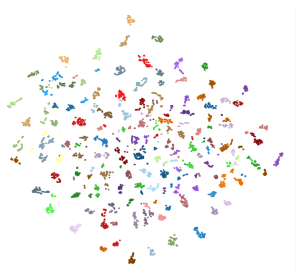
Fig. 15. t-SNE clustering of individual topological data for *Fear* emotion at different perplexities.



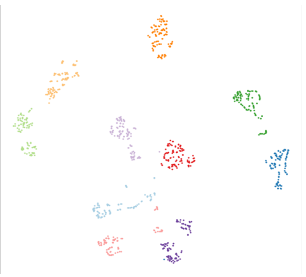
(a) *Happiness* perp.: 30 (full)



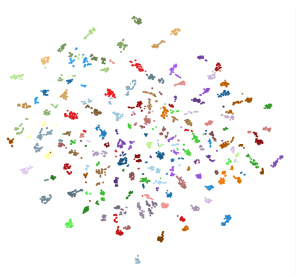
(b) *Happiness* perp.: 30 (10 subj.)



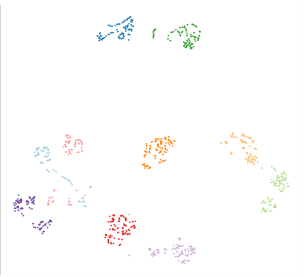
(c) *Happiness* perp.: 40 (full)



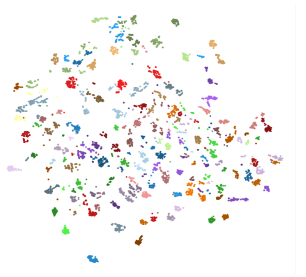
(d) *Happiness* perp.: 40 (10 subj.)



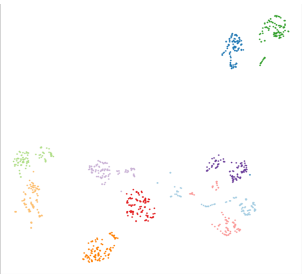
(e) *Happiness* perp.: 50 (full)



(f) *Happiness* perp.: 50 (10 subj.)

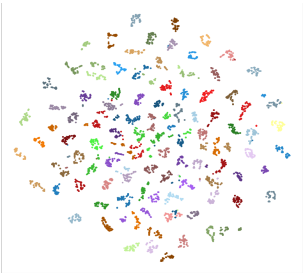


(g) *Happiness* perp.: 100 (full)

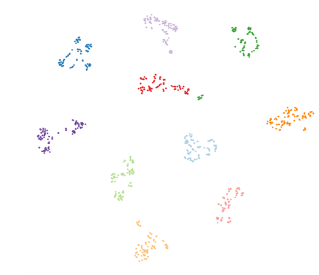


(h) *Happiness* perp.: 100 (10 subj.)

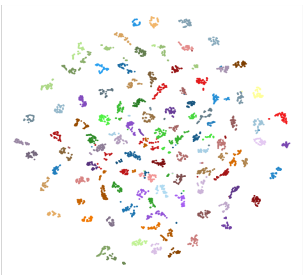
Fig. 16. t-SNE clustering of individual topological data for *Happiness* emotion at different perplexities.



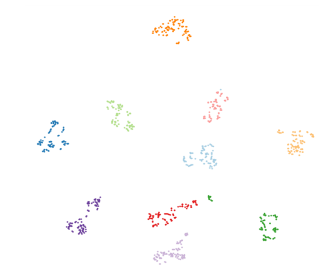
(a) *Sadness* perplexity: 30 (full)



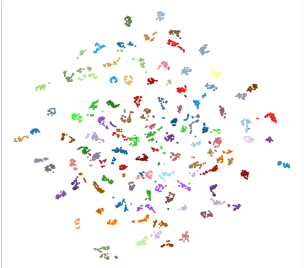
(b) *Sadness* perplexity: 30 (10 subj.)



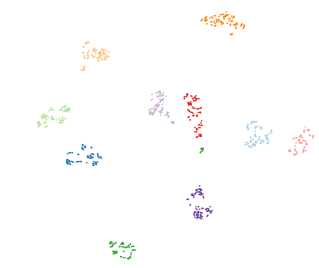
(c) *Sadness* perplexity: 40 (full)



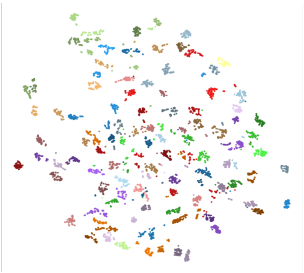
(d) *Sadness* perplexity: 40 (10 subj.)



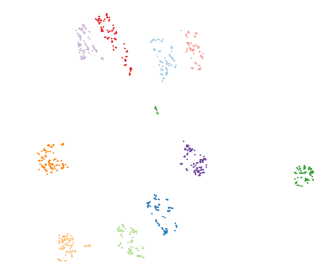
(e) *Sadness* perplexity: 50 (full)



(f) *Sadness* perplexity: 50 (10 subj.)

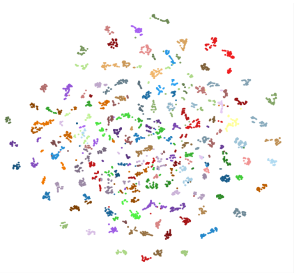


(g) *Sadness* perplexity: 100 (full)

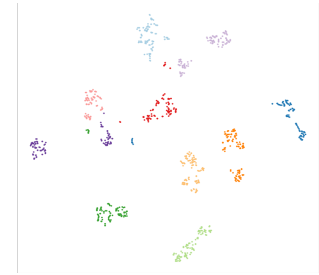


(h) *Sadness* perplexity: 100 (10 subj.)

Fig. 17. t-SNE clustering of individual topological data for *Sadness* emotion at different perplexities.

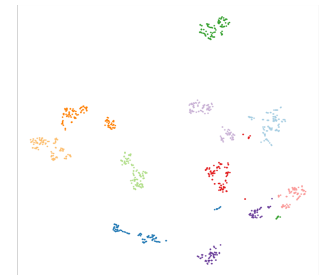


(a) *Surprise* perplexity: 30 (full)

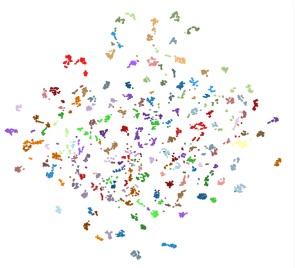


(b) *Surprise* perplexity: 30 (10 subj.)

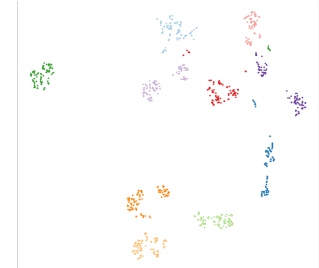
(c) *Surprise* perplexity: 40 (full)



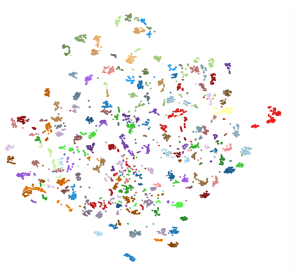
(d) *Surprise* perplexity: 40 (10 subj.)



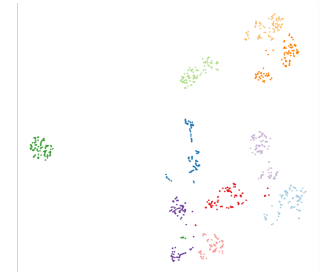
(e) *Surprise* perplexity: 50 (full)



(f) *Surprise* perplexity: 50 (10 subj.)

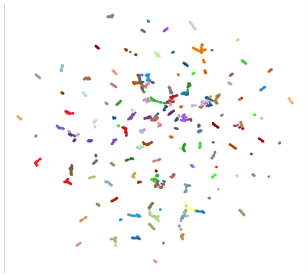


(g) *Surprise* perplexity: 100 (full)



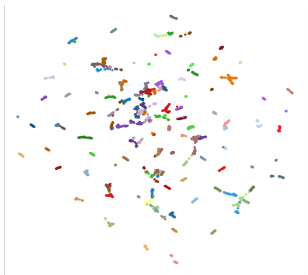
(h) *Surprise* perplexity: 100 (10 subj.)

Fig. 18. t-SNE clustering of individual topological data for *Surprise* emotion at different perplexities.



(a) *Anger* perplexity: 30 (full)

(b) *Anger* perplexity: 30 (10 subj.)



(c) *Anger* perplexity: 40 (full)

(d) *Anger* (10 subj.) Perplexity=40

(e) *Anger* perplexity: 50 (full)

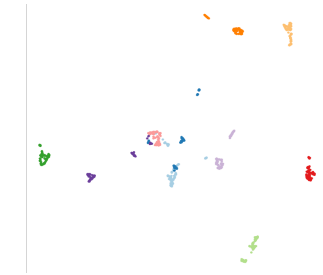
(f) *Anger* perplexity: 50 (10 subj.)

(g) *Anger* perplexity: 100 (full)

(h) *Anger* perplexity: 100 (10 subj.)

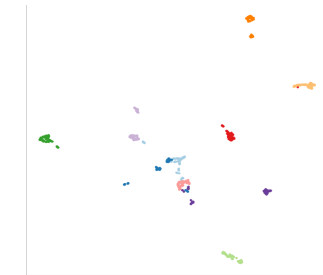
Fig. 19. UMAP clustering of individual topological data for *Anger* emotion at different perplexities.

(a) *Disgust* perplexity: 30 (full)



(b) *Disgust* perplexity: 30 (10 subj.)

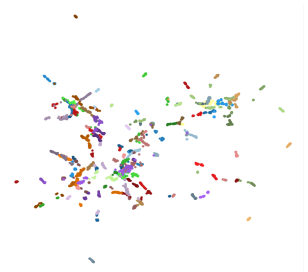
(c) *Disgust* perplexity: 40 (full)



(d) *Disgust* perplexity: 40 (10 subj.)

(e) *Disgust* perplexity: 50 (full)

(f) *Disgust* perplexity: 50 (10 subj.)



(g) *Disgust* perplexity: 100 (full)

(h) *Disgust* perplexity: 100 (10 subj.)

Fig. 20. UMAP clustering of individual topological data for *Disgust* emotion at different perplexities.

(a) *Fear* perplexity: 30 (full)

(b) *Fear* perplexity: 30 (10 subj.)

(c) *Fear* perplexity: 40 (full)

(d) *Fear* perplexity: 40 (10 subj.)

(e) *Fear* perplexity: 50 (full)

(f) *Fear* perplexity: 50 (10 subj.)

(g) *Fear* perplexity: 100 (full)

(h) *Fear* perplexity: 100 (10 subj.)

Fig. 21. UMAP clustering of individual topological data for *Fear* emotion at different perplexities.

(a) *Happiness* perp.: 30 (full)

(b) *Happiness* perp.: 30 (10 subj.)

(c) *Happiness* perp.: 40 (full)

(d) *Happiness* perp.: 40 (10 subj.)

(e) *Happiness* perp.: 50 (full)

(f) *Happiness* perp.: 50 (10 subj.)

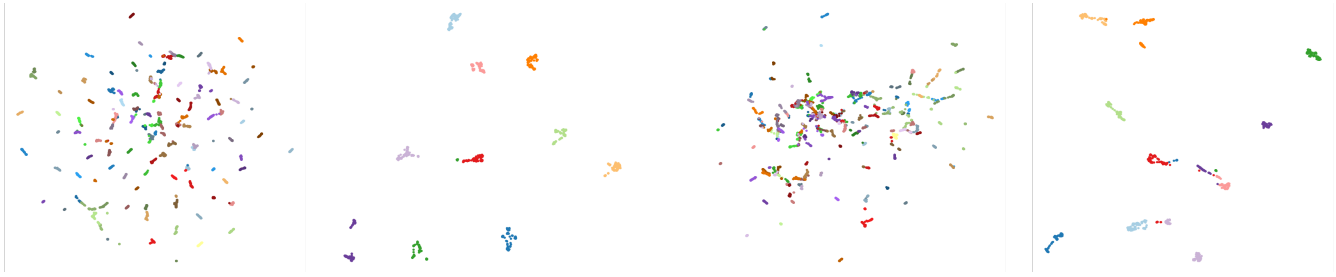
(g) *Happiness* perp.: 100 (full)

(h) *Happiness* perp.: 100 (10 subj.)

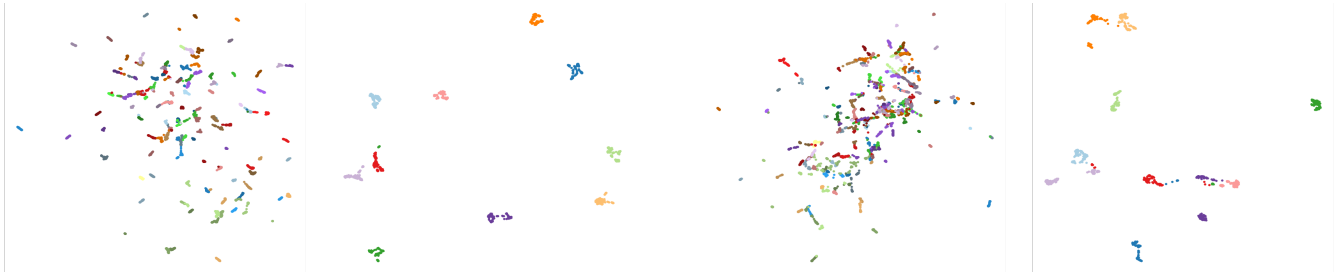
Fig. 22. UMAP clustering of individual topological data for *Happiness* emotion at different perplexities.

(a) *Sadness* perplexity: 30 (full) (b) *Sadness* perplexity: 30 (10 subj.)

(c) *Sadness* perplexity: 40 (full) (d) *Sadness* perplexity: 40 (10 subj.)



(e) *Sadness* perplexity: 50 (full) (f) *Sadness* perplexity: 50 (10 subj.)



(g) *Sadness* perplexity: 100 (full) (h) *Sadness* perplexity: 100 (10 subj.)

Fig. 23. UMAP clustering of individual topological data for *Sadness* emotion at different perplexities.

(a) *Surprise* perplexity: 30 (full) (b) *Surprise* perplexity: 30 (10 subj.)

(c) *Surprise* perplexity: 40 (full) (d) *Surprise* perplexity: 40 (10 subj.)

(e) *Surprise* perplexity: 50 (full) (f) *Surprise* perplexity: 50 (10 subj.)

(g) *Surprise* perplexity: 100 (full) (h) *Surprise* perplexity: 100 (10 subj.)

Fig. 24. UMAP clustering of individual topological data for *Surprise* emotion at different perplexities.