A Dynamic Curvature Based Approach for Facial Activity Analysis in 3D Space

Shaun Canavan, Yi Sun, Xing Zhang, and Lijun Yin

Department of Computer Science, State University of New York at Binghamton

Abstract

This paper presents a novel dynamic curvature based approach (dynamic shape-index based approach) for 3D face analysis. This method is inspired by the idea of 2D dynamic texture and 3D surface descriptors. The dynamic texture (DT) based approaches [30][31][32] encode and model the local texture features in the temporal axis, and have achieved great success in applications of 2D facial expression recognition. In this paper, we propose a so-called Dynamic Curvature (DC) approach for 3D facial activity analysis. To do so, the 3D dynamic surface is described by its surface curvature-based shape-index information. The surface features are characterized in local regions along the temporal axis. A dynamic curvature descriptor is constructed from local regions as well as temporal domains. To locate the local regions, we also applied a 3D tracking model based method for detecting and tracking 3D features across 3D dynamic sequences. Our method is validated through our experiment on 3D facial activity analysis for distinguishing neutral vs. non-neutral expressions, prototypic expressions, and their intensities.

Keywords: dynamic curvature, face analysis, 3D facial expressions, dynamic texture.

1. Introduction

Facial activity analysis using 3D videos has become an intensified research topic in recent years [14][27][28][29]. 3D representation of real life objects allows for a more realistic behavior analysis and understanding. However, it is difficult to process the data in a 3D dynamic space. The major challenges lie in the difficulties of 3D data registration, 3D feature extraction, and 3D data description. In this paper, we investigate approaches for effective 3D feature representations in order to characterize the dynamic geometric features across time for facial activity analysis.

Dynamic Texture (DT) is an effective method for appearance-based facial analysis from consecutive video-frames [30]. Some existing approaches to represent and extract dynamic textures were based on optical flow [34], motion history images [33], volume local binary patterns [32], and free form deformation [31]. Dynamic texture based methods have been successfully used for applications in facial expression recognition [32][33][34]. However, they are essentially 2D-based approaches with limitations of various imaging conditions (e.g., illuminations, poses, etc.).

Motivated by the dynamic texture approaches from 2D videos, we propose a new approach to describe the 3D facial activity in 3D videos, which is dynamic curvature in a 3D dynamic space for facial activity analysis. We segment the 3D facial meshes into several isolated local regions based on facial actions. Then, the histograms of shape-index from curvatures across multi-frame geometric surfaces are concatenated to form a unique descriptor - dynamic curvature for 3D facial behavior representation. Such a descriptor that represents the temporal dynamics of the facial surface will be input to a classifier (*e.g,* SVM) for further classification of facial activities (e.g., expressions, identities, etc.).

In order to segment the facial regions, it is critical to detect and track facial features across 3D geometric sequences. While research in 2D modality based tracking has produced a number of successful and widely used algorithms [10][35][36][9][11][4], research on 3D modality based analysis still faces the challenges of geometric landmark detection, mesh registration, motion tracking, and data representation. Traditionally, feature detection in 3D geometric space was performed by registration or 2D-to-3D mapping methods on static models [5][6][1][12][2][13][7][8]. In this paper, we apply a tracking model constructed from a temporal 3D point distribution for this task.

We will evaluate the performance through an application for facial activity classification: neutral vs. non-neutral; six prototypic expressions; and the status of expression activity in low intensity vs. in high intensity.

The rest of the paper is organized as follows: Section 2 provides a brief description of our tracking model. Section 3 describes dynamic curvature based 3D feature representation. Section 4 reports experiments and evaluations on the feature point detection and dynamic curvature classification for facial activity recognition. Finally, discussion and conclusion are given in Section 5.

2. 3D Shape Tracking Model

3D range data exhibits shapes of facial surfaces explicitly. This shape representation provides a direct match with the 3D active shape model due to its inherent and explicit shape representation in 3D space. We present a 3D shape tracking model to describe the shape variation across the 3D sequences.

To construct a shape model, we apply a similar representation of the point distribution model to describe the 3D shape, in which a parameterized model S is constructed by 83 landmark points on each model frame. An example of landmark points is shown in Figure 1. Such a set of feature points (shape vector) is aligned by a Procrustes analysis method [9]. Then the principal component analysis (PCA) is then performed on the new aligned feature vector. This is done to estimate the different variations of all the training shape data. To do so, each shape deviation from the mean and the covariance matrix are calculated, resulting in the modes of variation, V, of the training shapes along the principal axes. Given V and a vector of weights, w, that controls the shape, we can approximate any shape from the training data by:

$$S = \bar{s} + Vw \tag{1}$$

The vector of weights, w, allows us to generate new samples by varying its parameters within certain limits.

When approximating a new shape S, the point distribution model is constrained by both the variations in shape and the shapes of neighbor frames. Figure 1 shows an example of the shape model and the tracked 83 feature points. The detailed algorithm is described in [37].

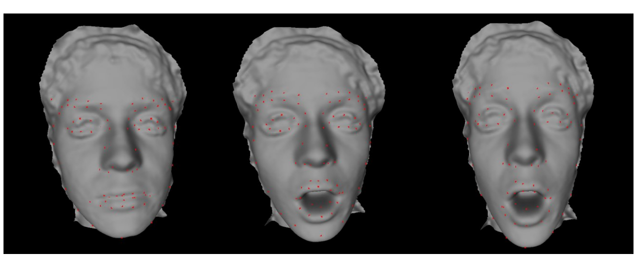

Figure 1: Example of tracked 83 feature points on a surprise expression sequence.

3. Dynamic Curvature Based Approach

Given the detected facial features and the resulting local regions, the shape (curvature) change along the 3D model sequences can be observed in individual local regions. Inspired by the recent work on facial analysis from static curvature based approaches [2] and dynamic texture based approaches [31][32], we propose a so-called Dynamic Curvature based descriptor for facial activity classification. Visual texture of an object is the reflection of its physical surface and lighting reflectance. Physical surface of an object can be characterized by its surface descriptor, e.g., primitive curvature type, shape-index, normal, etc. Given this observation, we extend the concept of Dynamic Texture in 2D space to the concept of Dynamic Curvature in 3D space (Dynamic Shape-Index). Unlike dynamic texture based approaches, which require building a rotation/scale invariant vector for feature representation, we use 3D shape descriptors (e.g., primitive curvature types, shape index) as our feature representation. Curvature is a good representation of local surface geometric characteristics. It is invariant to affine transformation like shift or rotation. Facial surface change reflects facial expression change. Encoding the surface changes of local facial region using dynamic curvature representation, we are able to capture the temporal dynamics of facial surface for expression classification.

After the model regions have been localized, the regional shape is described and quantified by curvature based shape-index. The dynamic curvature descriptor is then generated for classification.

3.1. Shape description and quantization

Shape index is a quantitative measure of the shape of a surface at a point [15][16]. It gives a numerical value to a shape thus making it possible to mathematically compare shapes and categorize them. Shape Index is defined as follows:

$$S = \frac{2}{\pi} \times \arctan\left(\frac{k2 + k1}{k2 - k1}\right) \tag{2}$$

where k_1 and k_2 are the principal (minimum and maximum) curvatures of the surface, with $k_2 >= k_1$. With this definition, all shapes can be mapped on the range *[-1.0, 1.0]*. Every distinct surface shape corresponds to a unique shape index value. The shape index is computed for each point on the model. We use a cubic polynomial fitting approach to compute the eigen-values of the Weingarten Matrix [15], resulting in the minimum and maximum curvatures (k_1, k_2). The shape index scale is normalized to *[0, 1]*, and encoded as a continuous range of grey-level values between 1 and 255. To quantify the curvature based measurement for an efficient description of a model, we transform the shape index scale to a set of nine quantization values from concave to convex, namely (1) Cup (0); (2) Trough (0.125); (3) Rut Saddle (0.25); (4)

Rut (0.375); (5) Saddle (0.5); (6) Saddle Ridge (0.625); (7) Ridge (0.75); (8) Dome (0.875); and (9) Cap (1), as shown in Figure 2.

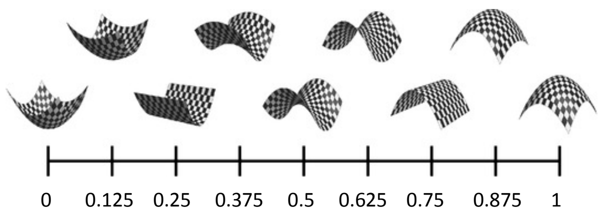

Figure 2: Shape index quantization to nine values: Cup (0), Trough (0.125), Rut Saddle (0.25), Rut (0.375), Saddle (0.5), Saddle Ridge (0.625), Ridge (0.75), Dome (0.875), and Cap (1).

3.2. Dynamic Curvature Based Descriptor

Until this stage, each vertex on the 3D face model has been assigned a curvature-based label (*i.e.,* quantized shape index) based on the above shape analysis. Since each facial model is segmented into 8 sub-regions (e.g., eyes, nose, mouth, cheek, etc. as shown in Figure 3) from the set of tracked feature points, we are able to get the curvature distribution of each sub-region and combine them into a vector. To do so, we construct following histograms to form a dynamic curvature descriptor:

(1) *Regional Histogram of Intra-frame*: Given k facial frames and n regions for each individual frame, the histogram of shape-index of each region i of individual frame j is derived to form a histogram vector, h_i^j , where $i=1,\ldots n; j=1,\ldots k;$

$$h_i^j = [\tfrac{c_1}{c}, \tfrac{c_2}{c}, \ldots., \tfrac{c_9}{c}] \tag{3}$$

where c is the total number vertices of a local region i in a single frame j, and $c_1, \ldots\ c_9$ are the numbers of vertices with shape-index scale $1,\ldots 9$ in that region, respectively.

(2) *Regional Histogram of Inter-frame*: In each region i, the statistics of shape-index is counted in all k *frames* as a whole to form a second histogram vector, $\overline{h}_i^k$, where $i=1,\ldots n; j=1,\ldots k.$

$$\overline{h}_i^k = [\tfrac{C_1}{C}, \tfrac{C_2}{C}, \ldots., \tfrac{C_9}{C}] \tag{4}$$

where C is the total number vertices of a local region i across all k frames, and $C_1, \ldots\ C_9$ are the numbers of vertices with shape-index scale $1,\ldots 9$ in that region of all k frames, respectively.

(3) *Local Temporal Histogram*: For each sub-region i, we concatenate the histogram h_i^j across k frames along the temporal axis and the histogram $\overline{h}_i^k$ to formulate a local temporal histogram vector,

$$H_i^k = [h_i^1, h_i^2 \ldots, h_i^k, \overline{h}_i^k] \tag{5}$$

(4) *Global Temporal Histogram - Dynamic Curvature Descriptor*: For the facial model across k frames, we combine all the local temporal histograms of n regions to generate a global descriptor (so-called dynamic curvature descriptor), which will be used for subsequent classification,

$$H_D^k = [H_1^k, H_2^k \ldots, H_n^k] \tag{6}$$

where n is the number of local regions and k is the number of frames ($n=8$ and $k=3$ in this implementation).

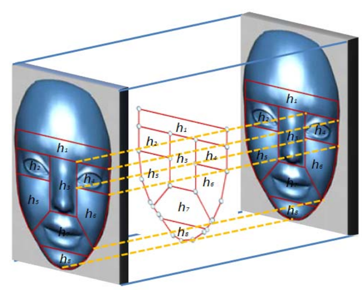

Figure 3: Illustration of Dynamic Curvature descriptor based on eight local regions.

3.3. Classification

After the dynamic curvature descriptor is created for 3D video sequences, we apply LDA for dimension reduction, and then use Support Vector Machine (SVM) classifiers to learn predictive power. Traditional SVM is used for binary classification. How to effectively extend it for multi-class classification problem is still an on-going research issue. One efficient way is to construct a multi-class classifier by combining several binary classifiers. The one-against-all SVM is constructed for each class by discriminating that class against the remaining M-1 classes. The number of SVMs used in this approach is M. A test pattern x is classified by using the winner-takes-all decision strategy, i.e., the class with the maximum value of the discriminant function $f(x)$ is the class that x belongs to.

Alternatively, the one-against-one SVM method is also known as one-versus-one method. An SVM is constructed for every pair of classes by training it to discriminate the two classes. Thus, the number of SVMs used in this approach is $M(M-1)/2$. A max-min strategy is used to determine the class that a test sample belongs to. That is to say, the class with the maximum number of votes for the test sample is assigned to the sample.

There are other existing multiclass SVM algorithms, e.g., directed acyclic graph SVM (DAGSVM) [17][18], Weston's multi-class SVM [19], and Crammer's multi-class SVM [20]. However, considering the algorithm complexity and classification performance, we chose the one-against-all SVM for the classification task.

4. Experiments and Evaluation

4.1. Database

A public database 4DFE [3] is used for our test. This is a 3D dynamic face model database, which contains 3D video sequences of six prototypic expressions of subjects. Each clip has neutral expressions and posed non-neutral expressions.

4.2. Facial Activity Classification

Inspired by the 2D dynamic texture based approach which is capable of distinguishing different expressions, we extend the concept to dynamic curvature based approach for handling 3D dynamic range model videos. One of the advantages is that the curvature based descriptor encodes the local surface shape information explicitly, thus being relatively robust with noise and pose changes. To verify such a new descriptor, we performed experiments on facial activity on three levels. First, we distinguish the facial activity by expressive face (with non-neutral expressions) and non-expressive face (with neutral appearances). Second, given the expressive face category, we apply the SVM (one-against-all) to classify the six prototypic expressions. Third, we further identify the intensity of each prototypic expression: either low intensity or high intensity.

We used 60 subjects from 4DFE for our experiment. The experiment is subject-independent. We randomly choose 50 subjects for training and 10 subjects for testing. Based on the tenfold cross-validation approach, by which the tests are executed 10 times with different partitions to achieve a stable generalization recognition rate. The classifier used for all three-level experiments is the two-class SVM. Followings are the results for three-level facial activity classification.

First Level: *Neutral vs. Non-Neutral.*

The confusion matrix is listed as below in Table 1. The average recognition rate to separate neutral with non-neutral expression is as high as 94.7%.

Table 1: Recognition rate for neutral/non-neutral expression

True\Estimate	Neutral	Non-Neutral
Neutral	95.1%	4.9%
Non-Neutral	5.7%	94.3%

Second Level: *Six prototypic expressions*

From the non-neutral group of video segments, we further classify six prototypic expressions: anger, disgust, sadness, happiness, fear, and surprise. The confusion matrix of distinguishing six universal expressions is listed in Table 2. The average recognition rate is 84.8%

Table 2: Recognition rate for six universal expressions (%)

True*Estimate*	*Anger*	*Disgus*	*Fear*	*Happy*	*Sad*	*Surpri*
Anger	83.6	5.5	4.9	0	3.8	2.2
Disgust	5.1	83.2	5.8	0	3.3	2.6
Fear	1.7	3.2	81.3	7.5	4.2	2.1
Happy	1.1	2.1	0	92.1	0	4.7
Sad	4.2	8.6	9.2	0	78	0
Surprise	1.1	1.9	3.6	3.9	0	89.5

Third Level: *Low Intensity vs. High Intensity*

For each recognized expression, their corresponding 3D video segments are further classified by the binary SVM for separating their degree of the expression: low intensity or high intensity. Below are the summary of the average rate (Table 3) and the individual confusion matrix (Table 4).

Table 3: Average separation rate of low/high intensities

Angry	Disgust	Fear	Happy	Sad	Surprise
80.6%	83.4%	79.1%	91.2%	78.4%	90.7%

Table 4: Confusion matrix of individual expression for intensity (low/high) separation

Expression	True\Estimate	Low	High
Angry	Low	81.8%	18.2%
	High	20.6%	79.4%
Disgust	Low	81%	19%

	High	14.2%	85.8%
Fear	Low	80.1%	19.9%
	High	21.9%	78.1%
Happy	Low	86.1%	13.9%
	High	3.7%	96.3%
Sad	Low	79.4%	20.6%
	High	23.6%	77.4%
Surprise	Low	85.5%	14.5%
	High	4.1%	95.9%

Observed from above results, the expression intensity of happiness and surprise is relatively easier to separate than the others due to their physically large movements of mouth and eyes, while sadness, fear, and angry have relatively small movements of these areas.

4.3. Comparison

We also conducted experiments with both our dynamic curvature based approach and other methods for recognizing expressions with both high and low intensities, respectively. We choose the recent and classic work for comparison, including 3D dynamic HMM [13][23], 3D dynamic Motion Units [23], 3D static surface primitive feature distribution [2], 2D dynamic motion units [22], 2D dynamic texture [32], and 2D static Gabor Wavelet [21]. As shown in the Table 5, the dynamic curvature based approach outperforms other approaches in both cases of low intensity and high intensity of expressions. Its performance is close to the 3D dynamic HMM based approach where spatial-temporal features were described in the HMM framework.

Table 5: Recognition rates from low intensity (LI) expressions and high intensity (HI) expressions using different approaches, respectively.

Methods	Low (LI)	High (HI)
3D dynamic curvature (our approach)	*75.1%*	*86.3%*
3D dynamic (HMM) [13][23]	72.4%	83.7%
3D dynamic (MU based [23]	57.3%	72.1%
3D static (PSFD) [2]	52.8%	71.7%
2D dynamic (MU based) [22]	56.6%	69.2%
2D dynamic (DT based) [32]	70.8%	81.5%
2D static (Gabor) [21]	50.4%	68.6%

5. Conclusion and Future Work

In this paper, we presented a new 3D feature representation using a so-called dynamic curvature based approach for facial activity analysis.

The experiments have shown the feasibility of such a new descriptor for 3D facial activity analysis. We have evaluated and validated its utility for dynamic curvature based expression classification in terms of neutral vs. non-neutral, various prototypic expressions, and their low/high intensities.

In the future work, we plan to develop a more robust method for estimating the direction of motion for the landmarks, including 3D edge information based on differences between vertex normal values. We will also validate our method through the application of 3D facial action unit detection and segmentation of dynamic expression sequences. The proposed 3D Dynamic Curvature based approach is in principle applicable (or extendible) to any other objects with 3D/4D mesh representation. Our future work will also include the evaluation on 3D feature detection and Dynamic Curvature descriptor on spontaneous expression data and other databases, such as [24][25][26].

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